Significance of Unsteady Thickness Noise Sources

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The broadband noise from rotors is often considered to be dominated by unsteady lift and trailing edge noise, but the predicted levels from these source mechanisms have been found to underestimate the observed noise levels in the plane of the rotor. This paper considers the application of Hawkings' "unsteady thickness noise" theory to account for this discrepancy. A generalized formulation for the sound radiation from a blade passing through homogeneous turbulence is derived and the theoretical results are compared with the measurements of noise from a model scale rotor presented by Paterson and Amiet.

Nomenclature

a_1, a_2	= coefficients of blade thickness distribution
b	= blade span, b_0 coefficient of blade thickness function
c	= blade chord
c_0	= speed of sound
$E(k)$	= energy spectrum of turbulence
$g(\ )$	= unsteady lift response function
$H(y)$	= surface of airfoil $y_3 = \pm H(y)$
h	= blade thickness = $2H$
K	= $\omega c/U$
k	= wavenumber of gust
$\hat{k}_2$	= $\frac{1}{2}(k_2 + \omega x_2/r_0 c_0)b$
k_e	= wavenumber of energy-bearing eddies
L	= unsteady lift or lengthscale of turbulence
M	= U/c_0
M_r	= convection Mach number
$\hat{n}$	= unit vectors normal to airfoil surface
p	= acoustic pressure
r	= distance from source to observer at correct retarded time
S	= surface of airfoil
$S_{pp}(\boldsymbol{x},\omega)$	= power spectrum of acoustic pressure at $\boldsymbol{x}$
t	= observer time
U	= mean flow velocity
u	= rms turbulence velocity
v	= induced velocity by sources
w	= velocity of gust
$\boldsymbol{x}$	= observer location
y	= coordinate system of airfoil
δ_{ij}	= Kronecker delta function
τ	= source time, τ_c – thickness/chord ratio
ρ_0	= density of fluid
Σ	= planform of airfoil
ϕ_{ij}	= wavenumber spectrum of turbulence
ω	= angular frequency
ζ	= coordinate system of gust
χ_i	= blade response functions

I. Introduction

THE broadband noise from rotors is often considered to be dominated by unsteady lift and trailing edge noise, but the predicted levels from these source mechanisms have been found to underestimate the observed noise levels in the plane of the rotor.[1] In some formulations of trailing edge noise (see for example, Howe[2]), the directionality would have been expected to peak in the direction of blade motion. However, these model the blade as a semi-infinite flat plate, which is only realistic at very high frequencies. A more general model is given by Amiet,[3,4] and this shows that for blades of finite chord, the directionality of trailing edge noise is a minimum in the direction of blade motion at the frequencies of practical interest. This is illustrated in Fig. 1, which shows the directionality of trailing edge noise from a blade with a chord to wavelength ratio of three. To account for measured levels it is therefore necessary to include the noise from drag dipoles, the directionality of which should peak in the direction of blade motion and hence dominate the noise radiated in the plane of the rotor.

The drag dipole strength is proportional to the fluctuating drag forces; these may be attributed to either the reaction of the blade to unsteady inflows or to regions of separated flow. The contribution from unsteady inflows is difficult to estimate directly since all models of the blade response to turbulence only consider infinitely thin airfoils, for which the two-dimensional instantaneous total drag is zero. Hawkings[5] has considered this problem and by reformulating the acoustic analysis has developed a prediction in terms of the thickness distribution of the blade. This mechanism is known as "unsteady thickness noise" and may be used to replace the drag dipole in the Ffowcs-Williams and Hawkings equation. Hawkings' unsteady thickness noise theory will be rederived here, and its significance relative to the noise from unsteady lift will be assessed. Since both of these source mechanisms depend on the inflow turbulence, a generalized form of the prediction formulae for these two mechanisms will be given. This applies to blades of finite span in linear motion. The power spectral density of the radiated acoustic pressure is also given in terms of the wavenumber spectrum of the inflow turbulence and blade response functions.

The application of this theory to rotating blades is considered in Sec. IV, and the results are compared with the experimental data presented in Ref. 1 for a rotor operating in a high level of turbulence. The comparisons are very encouraging and demonstrate the importance of the unsteady thickness noise mechanism.

II. Unsteady Thickness Noise

In the classical theory for sound radiation from an airfoil in turbulent flow, the acoustic sources are modeled by lift dipoles whose strength can be obtained from the response of the aerofoil to an upwash gust. This response is evaluated by modeling the airfoil as a flat plate of zero thickness and solving for the boundary condition that the flow normal to the plate should be zero. Hawkings[5] pointed out that to move

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the boundary condition to the actual surface of the airfoil, an additional monopole source distribution has to be included, and that this can dominate the radiated sound field in the plane of motion. Here Hawkings' theory is rederived, since the publications of this theory are not readily available.

Consider the surface of the airfoil to be described by the function $\pm H(y)$, where y is a set of blade-based coordinates (as illustrated in Fig. 2) being convected with uniform velocity $-U$ in the y_1 direction through a stationary gust. The velocity perturbation of the gust is $w(\zeta)$ relative to the stationary frame of reference, and ζ is a set of coordinates in this frame. A distribution of acoustic sources may be placed on the $y_3=0$ axis, giving an induced velocity $v(y)$ in the field so that the boundary condition $(U+w+v)\cdot\hat{n}=0$ is obtained on $y_3=\pm H$ at any fixed instant in time (where $\hat{n}$ is the unit vector normal to the surface). The term U describes the boundary condition due to the steady motion of the blade, and results in "steady thickness noise." Here we are concerned with the unsteady components, and so the U term may be dropped from the analysis and included in the final result, if required, using well known steady thickness noise formulations.

The boundary condition for the unsteady sources may be written as

$$(w_1+v_1)n_1+(w_2+v_2)n_2+(w_3+v_3)n_3=0 \text{ on } y_3=\pm H \quad (1)$$

Making the assumptions of thin airfoil theory and taking $H(y)$ to be small, the third term in this equation which describes the velocity normal to the $y_3=0$ plane may be expanded using Taylor's theorem as

$$(w_3+v_3)|_{y=\pm H}=(w_3+v_3)|_{y_3=0} \pm H(y)\frac{\partial}{\partial y_3}(w_3+v_3)|_{y_3=0}+\ldots \quad (2)$$

The response velocity v is considered in two parts so that $v=v^{(1)}+v^{(2)}$, where $v^{(1)}$ is the solution to the classical unsteady lift problem $(w_3+v_3^{(1)})=0$ on $y_3=0$, and $v^{(2)}$ is the correction required to move the boundary condition to the surface $y_3=\pm H(y)$. The remaining terms on the right of Eq. (2) are

$$v_3^{(2)}|_{y_3=0}\pm H(y)\frac{\partial}{\partial y_3}(w_3+v_3^{(1)}+v_3^{(2)})|_{y_3=0}$$

$$=v_3^{(2)}|_{y_3=\pm H}\pm H(y)\frac{\partial}{\partial y_3}(w_3+v_3^{(1)})|_{y_3=0}$$

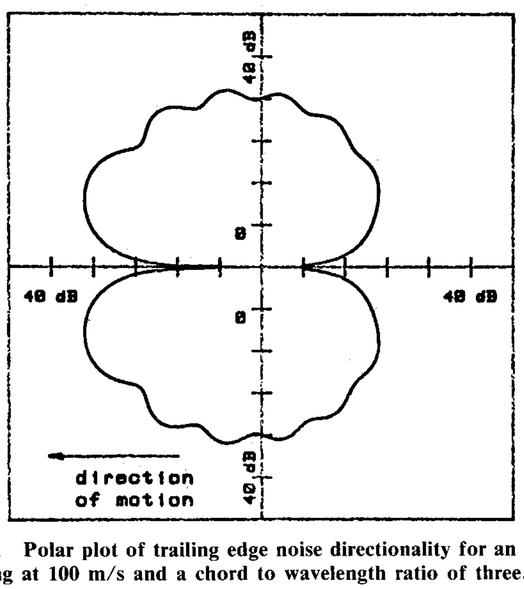

Fig. 1 Polar plot of trailing edge noise directionality for an airfoil moving at 100 m/s and a chord to wavelength ratio of three.

Thus the boundary condition [Eq. (1)] may be rearranged to yield

$$v^{(2)}\cdot\hat{n}|_{y_3=\pm H}=-(w_1+v_1^{(1)})\cdot n_1|_{y_3=\pm H} -(w_2+v_2^{(1)})\cdot n_2|_{y_3=\pm H}-H(y)\frac{\partial}{\partial y_3}(w_3+v_3^{(1)})|_{y_3=0}\cdot n_3$$

where n_3 is evaluated on $y_3=\pm H$.

Since $v^{(1)}$ can be generated by an array of dipoles on $y_3=0$, it follows that $v_1^{(1)}$ and $v_2^{(1)}$ are odd functions of y_3 and $v_3^{(1)}$ is a maximum at $y_3=0$. Hence, by using a Taylor's series expansion on the first two terms, we obtain

$$v^{(2)}\cdot\hat{n}|_{y_3=\pm H}=-\left(w_1n_1+w_2n_2+H(y)\frac{\partial w_3}{\partial y_3}n_3\right)\Big|_{y_3=0} \pm H\frac{\partial}{\partial y_3}\left((w_1+v_1^{(1)})\cdot n_1+(w_2+v_2^{(1)})\cdot n_2\right)\Big|_{y_3=0} +\ldots 0\left(H^2\frac{\partial^2}{\partial y_3^2}(w+v)\cdot\hat{n}\right)\Big|_{y_3=0} \quad (3)$$

The remainder terms in this expansion are of order $(h/L_3)^2$, where L_3 is the lengthscale of the turbulence normal to the chord and will be frequency-dependent. In most applications, $H \ll L_3$, and so the higher-order terms may be ignored. However, at very high frequencies this assumption may not apply, and higher-order terms would have be included. The perturbation velocity $v^{(2)}$ may therefore be obtained by placing an array of monopoles and dipoles on the axis $y_3=0$. The dipole distribution is given by the second term in Eq. (3), and represents a small correction to the classical problem necessary to move the boundary condition from $y_3=0$ to $y_3=\pm H$. The first term in Eq. (3) can be matched by a monopole distribution of the type

$$p(x,t)=\frac{+1}{4\pi}\rho_0\frac{\partial}{\partial t}\int_S\left[\frac{v^{(2)}\cdot\hat{n}}{r|1-M_r|}\right]dS$$

where S is both the upper and lower surface of the aerofoil. Thus, if Σ is the planform of the blade on $y_3=0$ (see Fig. 2),

$$dS=\left[1+\left(\frac{\partial H}{\partial y_1}\right)^2+\left(\frac{\partial H}{\partial y_2}\right)^2\right]^{1/2}d\Sigma$$

giving

$$p(x,t)=\frac{-1}{2\pi}\rho_0\frac{\partial}{\partial t}\int_\Sigma\left[\frac{w_1n_1+w_2n_2+H(y)(\partial w_3/\partial y_3)n_3}{r|1-M_r|}\right] \times\left[1+\left(\frac{\partial H}{\partial y_1}\right)^2+\left(\frac{\partial H}{\partial y_2}\right)^2\right]^{1/2}d\Sigma$$

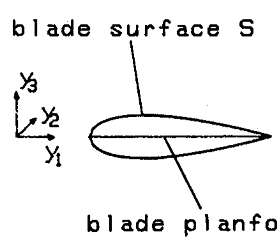

Fig. 2 Coordinate system of blade: upper surface $y_3=H(y)$, lower surface $y_3=-H(y)$.

If we limit consideration to uniformly tapered blades, then the unit vectors n_1, n_2, and n_3 may be evaluated as

$$n_1 = -\frac{\partial H}{\partial y_1}\left[1+\left(\frac{\partial H}{\partial y_1}\right)^2+\left(\frac{\partial H}{\partial y_2}\right)^2\right]^{-1/2}$$

$$n_2 = -\frac{\partial H}{\partial y_2}\left[1+\left(\frac{\partial H}{\partial y_1}\right)^2+\left(\frac{\partial H}{\partial y_2}\right)^2\right]^{-1/2}$$

$$n_3 = \left[1+\left(\frac{\partial H}{\partial y_1}\right)^2+\left(\frac{\partial H}{\partial y_2}\right)^2\right]^{-1/2}, \quad \text{on } y_3 = +H$$

Then, by assuming the turbulence to be incompressible so that $\mathrm{div}w = 0$, $\partial w_3/\partial y_3$ may be replaced by

$$-\frac{\partial w_1}{\partial y_1} - \frac{\partial w_2}{\partial y_2}$$

so we obtain

$$p(x,t) = \frac{+1}{4\pi}\rho_0\frac{\partial}{\partial t}2\int_\Sigma\left[\frac{\partial/\partial y_1(w_1H)+\partial/\partial y_2(w_2H)}{r\,|1-M_r|}\right]\mathrm{d}\Sigma$$

This is Hawkings' unsteady thickness noise result for an incompressible gust. It may be further simplified by replacing H by the actual blade thickness $h = 2H$ and considering the acoustic far field, where the derivatives may be taken outside the integral

$$p(x,t) = \frac{1}{4\pi}\frac{\partial^2}{\partial t\partial x_\beta}\int_\Sigma\left[\frac{\rho_0 w_\beta h}{r\,|1-M_r|}\right]\mathrm{d}\Sigma \quad \beta=1,2 \tag{4}$$

It is significant that this is not a monopole-type source, since there is no net mass displacement at any instant. This is shown by considering the integral of $v^{(2)}\cdot\hat{n}$ over the surface of the blade:

$$\int_S v^{(2)}\cdot\hat{n}\mathrm{d}S = \int_\Sigma \frac{\partial(Hw_2)}{\partial y_2}+\frac{\partial(Hw_1)}{\partial y_1}\mathrm{d}\Sigma$$

The element of area of the planform $\mathrm{d}\Sigma = \mathrm{d}y_1\mathrm{d}y_2$, so that by integrating the first term of the integrand over y_2 and the second over y_1, we obtain

$$\int_{-c_{max}/2}^{c_{max}/2} H[b(y_1),y_1]\cdot w_2\mathrm{d}y_1 + \int_{-b/2}^{b/2} H[y_2,c(y_2)]\cdot w_1\mathrm{d}y_2$$

where $b(y_1)$ and $c(y_2)$ specify the local span and the local chord respectively. However, the thickness H is zero at the edges of the blade so both of these integrands are zero , and hence there is no net monopole source strength.

A better interpretation of the unsteady thickness noise term is to consider it as equivalent to an unsteady drag force component. This may be illustrated by evaluating Eq. (4) for a harmonic inflow gust of the type $w = w_i\exp(ik_1\zeta_1 + ik_2\zeta_2 + ik_3\zeta_3)$, through which the blade is convected with uniform velocity U in the y_1 direction:

$$p(x,t) = \frac{1}{4\pi}\frac{\partial}{\partial x_\beta}\int_\Sigma\left[\frac{-i\omega\rho_0 w_\beta h e^{ik_1(y_1-U\tau)+ik_2y_2}}{r\,|1-M_r|}\right]\mathrm{d}\Sigma \tag{5}$$

where τ is source time and ω is the observed frequency $k_1U/(1-M_r)$.

Comparing this result with the acoustic field from a distribution of forces acting on the fluid shows that the $-i\omega\rho_0 hw_\beta$ term in the integrand is an equivalent observed drag of spanwise force per unit area. The integral of this

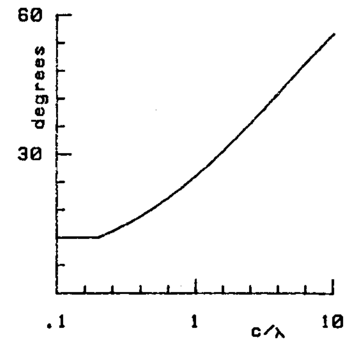

Fig. 3 Angle from rotor plane over which unsteady thickness noise dominates vs the chord to wavelength ratio.

over the planform radiates to the acoustic far field as an acoustic dipole-type source.

Unsteady thickness noise can be combined with unsteady lift noise to give a generalized formula for sound radiation from blades due to their interaction with inflow turbulence. This is obtained by considering the response of the blade to an unsteady upwash gust of the type used in Eq. (5). In general this may be written in the form

$$L(y_1,t) = \pi\rho_0 cUw_3 g(y_1,k)\exp[-ik_1U\tau]$$

where $g(\ )$ is the lift response function specified by Amiet.[6] Then by making the usual far-field approximations and replacing τ by

$$\left[t-\left(r_0-\frac{x_jy_j}{r_0}\right)\Big/c_0\right]\cdot(1-M_r)^{-1}$$

the far-field sound radiation from the blade due to a harmonic gust is specified as the sum of three terms, $j = 1,2,3$, defined as

$$p_j(x,t) = \frac{(i\omega/c_0)(x_j/r_0)}{4\pi r_0\,|1-M_r|}\exp[-i\omega(t-r_0/c_0)] \times\int_{\text{span}}\pi\rho_0 cUw_j\chi_j\exp\left[i\left(k_2-\frac{\omega x_2}{c_0r_0}\right)y_2\right]\mathrm{d}y_2$$

where

$$\chi_\beta = \frac{-i\omega}{\pi U}\int_0^c\left(\frac{h}{c}\right)\exp\left[\mathrm{i}\left(k_1-\frac{\omega x_1}{c_0r_0}\right)y_1\right]\mathrm{d}y_1, \quad \beta=1,2$$

$$\chi_3 = \int_0^c g(y_2,k)\exp\left[-i\left(\frac{\omega x_1}{c_0r_0}\right)y_1\right]\mathrm{d}y_1 \tag{6}$$

This is the principle result of this section, since it gives the complete sound field in terms of the inflow velocity field and the blade parameters. The lift response functions χ_3 are well documented and the thickness noise response χ_β can be obtained from the blade thickness distribution. This is often specified in terms of a power series of the type

$$\frac{h}{c} = 2\tau_c\left[b_0\sqrt{\frac{y_1}{c}}+a_1\left(\frac{y_1}{c}\right)+a_2\left(\frac{y_1}{c}\right)^2+a_3\left(\frac{y_1}{c}\right)^3+a_4\left(\frac{y_1}{c}\right)^4\right], \quad 0<y_1<c$$

where τ_c is the thickness to chord ratio and b_0 and a_n are nondimensional coefficients. Using this result, the preceding integral may be evaluated as

$$\chi_\beta = \frac{-i\omega c}{\pi U}\cdot 2\tau_c\left[\frac{b_0(\pi/2)^{1/2}}{(-iK)^{3/2}} + \frac{a_1}{(-iK)^2} + \frac{4a_2}{(-iK)^3} + \frac{18a_3}{(-iK)^4} + \frac{96a_4}{(-iK)^5}\right]$$

where

$$K = [k_1 - (\omega x_1/c_0 r_0)]c = \omega c/U$$

The relative significance of unsteady thickness noise compared with unsteady lift noise can be shown by considering the ratio

$$|\chi_\beta|^2 w_\beta^2 / |\chi_3|^2 w_3^2$$

In the frequency range of most practical interest (where $k_1 c/2 \gg 1$, $M \ll 1$, and $k_2 \ll \omega/c_0$), $|\chi_3|^2$ is approximated (see Amiet)[6] by

$$|\chi_3|^2 \approx (2\pi\hat{\omega})^{-1} \qquad M\hat{\omega} < \pi/4$$

$$\approx (2\pi\hat{\omega})^{-1}(2/\pi\hat{\omega}M) \qquad M\hat{\omega} > \pi/4$$

where $\hat{\omega}$ is the nondimensional frequency

$$\hat{\omega} = k_1 c/2 = [\omega c(1-Mr)/2U]$$

Hence,

$$\frac{w_\beta^2}{w_3^2}\frac{|\chi_\beta|^2}{|\chi_3|^2} = \frac{w_\beta^2}{w_3^2}(\tau_c b_0)^2 \times (1-Mr)\begin{cases} 1 & \hat{\omega}M < \pi/4 \\ (\pi\hat{\omega}M/2) & \hat{\omega}M > \pi/4 \end{cases}$$

It may be shown that $(w_1^2 + w_2^2)/w_3^2 \simeq 2$, b_0 is typically ~ 1.5, and the thickness to chord ratio $\tau_c \sim 0.1$. Therefore, the sound power from unsteady thickness noise is expected to be 13 dB less than the sound power from unsteady lift noise at low frequencies. However, due to their different directionalities, unsteady thickness noise will be significant at radiation angles that lie within ±12 deg of the rotor plane. At higher frequencies, where the blade chord is greater than a quarter of the acoustic wavelength, the relative significance of unsteady lift noise is reduced because of compressibility effects. This implies that unsteady thickness noise will be important at larger angles from the rotor disk plane. This effect is illustrated in Fig. 3, which shows the angle from the disk plane where unsteady thickness noise is more significant than unsteady lift noise as a function of c/λ. This demonstrates the importance of the mechanism, especially to helicopter applications where the observer angle to the rotor disk plane is small for most of the flyover.

At very high frequencies where the nondimensional parameter $H/L_3 \sim \omega h/U$ becomes of order one, the third-order terms in Eq. (3) cannot be ignored and may affect this result. However, this is not usually within the range of parameters that commonly apply to rotors of practical interest and so should not affect the results shown here.

III. Sound Radiation from an Airfoil in Homogeneous Isotropic Turbulence

The results of the previous section give the sound radiation from a blade moving through a harmonic gust. These may be extended to the case of a blade moving linearly through homogeneous isotropic turbulence by following the analysis given by Amiet.[6] Providing the turbulence is stationary and homogeneous, the power spectral density of the acoustic pressure in the far field is given as

$$S_{pp}(\boldsymbol{x},\omega) = \left(\frac{\omega}{4\pi r_0 c_0}\right)^2 (\pi\rho_0 cb)^2 U \times \int_{k_2}\int_{k_3} \frac{x_i x_j}{r_0^2}\chi_i\chi_j^*\phi_{ij}(\boldsymbol{k})\left[\frac{\sin(\hat{k}_2)}{\hat{k}_2}\right]^2 \mathrm{d}k_2\mathrm{d}k_3 \tag{7}$$

where $k_1 = \omega(1-Mr)/U$ and $\hat{k}_2 = ½(k_2 + \omega x_2/r_0 c_0)\cdot b$ and $\phi_{ij}(\boldsymbol{k})$ is the wavenumber spectrum of the turbulence.

This result can be further simplified by carrying out the integrals over k_2 and k_3. Since only the wavenumber spectrum $\phi_{ij}(\boldsymbol{k})$ depends on k_3, this integral may be evaluated exactly providing a suitable analytical approximation can be made for $\phi_{ij}(\boldsymbol{k})$. (Many different analytical models are available, for instance, the von Kármán spectrum, the Dryden spectrum, etc.) In general, for homogeneous, isotropic, and incompressible turbulence, the wavenumber spectrum will take the form

$$\phi_{ij}(\boldsymbol{k}) = \frac{E(|\boldsymbol{k}|)}{4\pi|\boldsymbol{k}|^2}\left(\delta_{ij} - \frac{k_i k_j}{|\boldsymbol{k}|^2}\right)$$

where δ_{ij} is the Kronecker delta function, equal to one when $i=j$ and zero when $i \neq j$. This shows that when i or $j=3$ and $i \neq j$, then $\phi_{ij}(\boldsymbol{k})$ is an odd function of k_3. Hence, the integral over k_3 between $\pm\infty$ will be zero. Consequently, sound radiated by the $i=3$ or lift components is uncorrelated with the $i=1,2$ or unsteady thickness noise components, and the cross terms $i,j=1,3$ or $2,3$ are zero.

The intergrals over k_2 and k_3 in Eq. (7) for the unsteady thickness noise components may be written in the form

$$\int_{k_2}\int_{k_3} \frac{E(|\boldsymbol{k}|)}{4\pi|\boldsymbol{k}|^2}|\chi_1|^2\left[\frac{x_1^2}{r_0^2}\frac{(k_2^2+k_3^2)}{|\boldsymbol{k}|^2} - \frac{2x_1x_2}{r_0^2}\frac{k_1k_2}{|\boldsymbol{k}|^2} \times \frac{x_2^2}{r_0^2}\frac{(k_1^2+k_3^2)}{|\boldsymbol{k}|^2}\right]\left[\frac{\sin(\hat{k}_2)}{\hat{k}_2}\right]^2 \mathrm{d}k_2\mathrm{d}k_3$$

The integral over k_2 is strongly influenced by the region where $\hat{k}_2$ is close to zero. In this region $|\boldsymbol{k}|^2 \approx (k_1^2 + k_3^2)$ with an error of order $(U/c_0)^2$.

It was argued by Amiet[6] that the integrand would be dominated by the $[\sin(\hat{k}_2)/\hat{k}_2]^2$ function, so providing the other terms do not vary significantly in the region $|\hat{k}_2| < 10$, they could be considered constant and taken outside the $\hat{k}_2$ integral, leaving

$$\int_{k_2}\left[\frac{\sin(\hat{k}_2)}{\hat{k}_2}\right]^2 \mathrm{d}k_2 = \frac{2\pi}{b}$$

This approximation of the integral over k_2 is valid providing $\omega b/U \gg 1$. However, to ensure convergence, care must be taken when considering the terms of order $(k_2/\hat{k}_2)^2$. Then, by assuming a suitable model for the energy spectrum of the turbulence, the k_3 integration may be carried out exactly. Here we assume a von Kármán spectrum and obtain

$$S_{pp}(\boldsymbol{x},w) = \left(\frac{\omega}{4\pi r_0 c_0}\right)^2 \rho_0^2 c^2 bU\cdot 2\pi^3\left[\frac{4}{9\pi}\frac{u^2}{k_e^2}\left(\frac{k_e U}{\omega}\right)^{8/3}\right] \times \left\{|\chi_3|^2\left(\frac{x_3}{r_0}\right)^2 + |\chi_1|^2\left[\left(\frac{x_2}{r_0}\right)^2\left((1-M_r)^2 + \frac{3}{8}\right) + \frac{3}{8}\left(\frac{x_1}{r_0}\right)^2\right]\right\} \tag{8}$$

where $k_e = (1.34L)^{-1}$, L = turbulence lengthscale, u^2 is the mean square turbulence velocity, and use has been made of the fact that $\chi_1 = \chi_2$.

This result gives the power spectrum of the acoustic pressure field radiated by a blade moving uniformly through homogeneous turbulence. Since at low frequencies both the blade response functions are inversely proportional to $\sqrt{\omega}$, the spectrum is expected to be proportional to

$$\left(\frac{\omega b}{U}\right)^{-1}\left(\frac{\omega L}{U}\right)^{-2/3}\cdot(u/U)^2$$

which shows only a weak dependence on turbulence lengthscale. This is important for prediction purposes because this is the parameter that is most difficult to estimate for many applications.

This result also includes the complete effect of cross correlation between the different components of the radiated noise. As has already been pointed out, the unsteady lift and unsteady thickness noise components are uncorrelated. It may also be shown that the cross correlation between the two components of unsteady thickness noise is given by the convection Mach number M_r in Eq. (8) This will therefore be insignificant in low-speed applications, and the three components will sum in an uncorrelated manner when $M_r=0$.

IV. Rotor Noise Predictions

The result given by Eq. (8) applies to a blade in rectilinear motion moving through homogeneous turbulence. To use this to predict the noise from a rotating blade, it is necessary to assume that the spanwise correlation lengthscale is short at the frequencies of interest so that the contributions from each spanwise station may be summed independently. Also, it is assumed for computational convenience that the turbulent eddies are completely convected through the rotor before they are chopped by a second blade, hence blade-to-blade correlation does not occur. This simplification limits the application of Eq. (8) to high-frequency broadband noise where the harmonic components in the acoustic spectrum are not present. This is not a severe limitation, since more detailed expressions to allow for all these effects could be obtained using the method given by Paterson and Amiet.[1] The total predicted level from the rotor is obtained by averaging the contributions from 20 azimuthal positions and 5 radial locations in the rotor plane for each blade by using Eq. (8), with correct allowance for retarded times on the analysis bandwidth.

The results of this prediction method have been compared with the experimental measurements reported in Ref. 1. This gives directionality data for a 0.76-m-diam rotor with 4 blades operating in an open-jet wind tunnel. The inflow turbulence to the rotor was controlled by using grids upstream of the rotor, and measurements of the turbulence lengthscale and rms turbulence velocity were reported.

The results in Figs. 4–7 show the predicted levels of both unsteady lift and unsteady thickness noise as a function of the angle to the rotor axis. It is seen that at angles ±20 deg from the rotor disk plane, the unsteady thickness noise component is significant at all frequencies considered. Moreover, estimates of the levels within ±10 deg of the rotor would be underpredicted by more than 10 dB if this component were

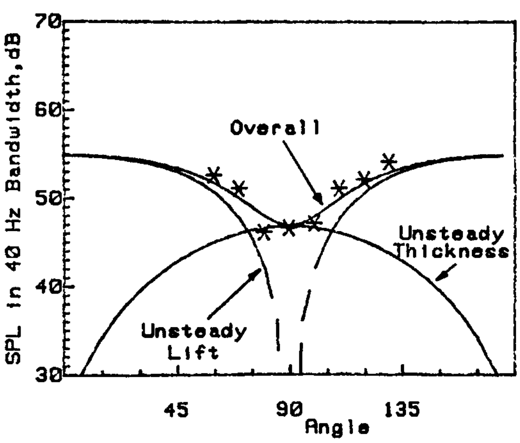

Fig. 4 Comparison of propeller noise directionality measurements with theoretical predictions at 4998 Hz.

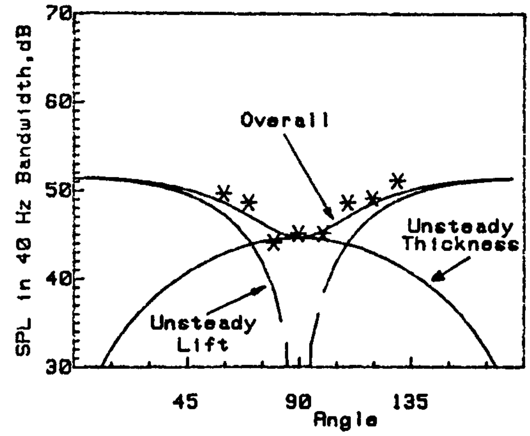

Fig. 5 Comparison of propeller noise directionality measurements with theoretical predictions at 6762 Hz.

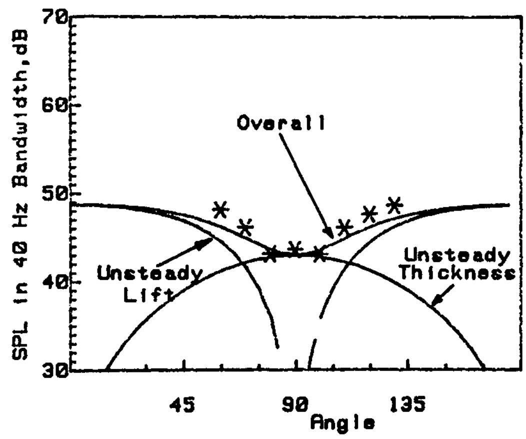

Fig. 6 Comparison of propeller noise directionality measurements with theoretical predictions at 8526 Hz.

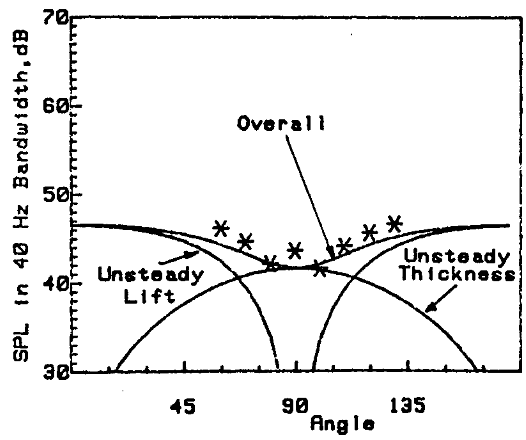

Fig. 7 Comparison of propeller noise directionality measurements with theoretical predictions at 10290 Hz.

not included. The fit to the data has been found to be remarkably good using these two source mechanisms. The contribution from trailing edge noise[3,4] was also predicted and not found to be important at these frequencies, although in many other applications it cannot be ignored.

These results have been obtained in an environment where the inflow turbulence was of a very high level; they demonstrate that in this situation, unsteady thickness and unsteady lift noise can be used to adequately describe the radiated sound level. However, in other applications where the turbulence is of a lower level and a much larger lengthscale, such good agreement has not been obtained.[7,8] Yet by arbitrarily adjusting the turbulence lengthscale, a good agreement with the measured data can be obtained, which suggests that the error in the predictions is due to the inadequacy of the turbulence model rather than to the prediction of the source mechanisms.

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